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COMMENT

Central charge for the half-filled Hubbard chain

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Abstract. Zero-temperature finite-size Bethe ansatz calculations of the central charge for the Hubbard chain are compared with the finite-temperature Bethe ansatz calculations of the same charge.

In their recent paper Itoyama and Thacker [1] raised a very interesting problem about the interrelationship between conformal symmetry and quantum integrability; while the first holds only at the critical temperature $T = T_c$, where the theory is massless, the second applies to the cases where $T \neq T_c$ and the spectrum of excitations has a mass gap. At the moment, although conformal invariance permits us to determine completely the correlation functions [2] and partition functions [3] at $T = T_c$ nothing is known about the systematic extension of these results for $T \neq T_c$.

The Hubbard model in one dimension represents an example of an exactly integrable model [4] which for the half-filled band case is believed to be critical [5]. Using the zero-temperature Bethe ansatz method adapted to the case of finite-size chains, Woynarovich and Eckle [5] were able to calculate the finite-size corrections to the ground-state energy which, in turn, permitted them to obtain the value of central charge $c = 1$ under the conditions that the self-interaction coupling constant $U > 0$, in which case the Hubbard model exhibits the same critical behaviour as the isotropic Heisenberg antiferromagnet. According to [5] the above analogy does not exist for $U \to 0$ due to the essential singularity of the model in this limit.

Here I would like to demonstrate that

(a) the value $c = 1$ can also be obtained from the finite-temperature Bethe ansatz calculations made for infinite chains [6],

(b) the value $c = 1$ holds for $U > 0$ and $U \to 0$.

According to Affleck [7] and Blöte *et al*/[8] the specific heat C and the central charge c are connected via the formula $C = \pi c T / 3v$ where $\hbar = \kappa_B = 1$, and v is the 'velocity of light' [7]. Takahashi [6] obtained $v = 2I_1(x)/I_0(x)$ where $I_\nu(x)$ is the modified Bessel function of order ν and $x = \pi/2U$. He also obtained $C/T = (\pi/6) I_0(x)/I_1(x)$. The results given permit us to define the charge c in the following way: $c = (C/T)3v/\pi$. This immediately produces $c = 1$ for all $U > 0$, in complete agreement with the results of [5]. In agreement with [5], he also noticed an essential singularity of the model at $U = 0$.

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